

Being, Nothing, Becoming Hegel and Us – A Formalization¹

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ABSTRACT: The article analyzes and formalizes the Hegelian categories of the second part of the Logic of Being, thus concluding this first project. Hegel's terminology is translated into 21st century English and the core passages are formalized. This article is the continuation of the first one published in this journal a year ago. It concludes the corrected and partially formalized commentary on the Logic of Being.

Key words: Hegel's Logic, Logic of Being, Formalization of Hegel's Logic.

RESUMO: Os autores analisam e formalizam as categorias hegelianas da segunda parte da Lógica do Ser, encerrando, assim, este primeiro projeto. A terminologia hegeliana foi transposta para o português do século XXI e as passagens centrais foram formalizadas. Este artigo é a continuação do primeiro, publicado nesta revista um ano atrás, e leva a fim e cabo o comentário corrigido e parcialmente formalizado da Lógica do Ser.

Palavras-chave: Lógica de Hegel, Lógica do Ser, Formalização da Lógica de Hegel.

Chapter 3 – Being-for-itself, the One and the Many, Repulsion and Attraction⁴

1 1 3 Being-for-itself

The first book of Hegel's *Science of Logic* discusses the Logic of Being, the second one the Logic of Essence and the third one the Logic of the Concept. Each

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book is divided into three sections (*Abschnitte*), and each section (Quality, Quantity, Measure) in its turn is subdivided into three chapters (*Kapitel*). – The first (Being, Nothing, Becoming) and the second (*Dasein*, Finitude, Infinitude) chapter of the first part of the book, the Logic of Being, have already been dealt with. To complete this first part, whose main topic is Quality, we have to discuss the chapter on Being-for-itself, the One and the Many, Repulsion and Attraction.

In *Being-for-itself the qualitative Being is fulfilled*; it is infinite Being. The Being of the beginning is indeterminate. *Dasein* is sublated (*aufgehoben*), but only immediately sublated Being; thus it contains only the first negation, which is itself immediate; Being is equally preserved, and both [Being and the first Negation] are united in *Dasein* in a simple unity. But precisely for this reason they are still *different* from each other and their unity is *not* [dialectically] *posited* yet. *Dasein*, is therefore, the sphere of difference, of dualism, the field of finitude. Determinateness is determinateness as such, a relative, non-absolute being-determinate. In Being-for-itself the difference between Being and Determinateness or negation is posited and conciliated; Quality, Otherness, Limit, as well as Reality, Being-in-itself, Ought etc. are the imperfect formations of Negation in Being on which the difference between both [Being and Being-in-itself] is still based. But insofar as in Finitude the Negation became Infinitude, the *posited* Negation of Negation, it is simple relation to itself and thus in itself the conciliation (*Ausgleichung*) with Being – *absolute being-determinate*. (Hegel, *Wissenschaft der Logik*. Frankfurt a.M.: Suhrkamp, 1969, vol. 1, p. 174).

This long quotation of Hegel's text will be our point of reference for the interpretation, in order to understand the transition from Being (1 1 1) to *Dasein* (1 1 2), from *Dasein* to Good Infinitude (1 1 2 C) and from Good Infinitude to Being-for-itself (1 1 3 A a) and to the issue of the One and the Many (1 1 3 A b).

1 1 3 A Being-for-itself as such

Being-for-itself, as discussed above, is *Dasein* with Good Finitude and Good Infinitude conceived again as something positive: it is the negation of negation. Through the first negation determinate *Dasein* emerges from indeterminate Being, and determinate *Dasein* proves to be Good Finitude and Good Infinitude. Through the second negation, the negation of negation, we leave the multiplicity of poles that delimit and determine each other in *Dasein* and return to the unity that is Being-for-itself. Thus every Being-for-itself is a Being for One, viz., Being for a One.

Let's now translate, step by step, the heavy Hegelianism of the above quotation into English.

First step: The Being of the beginning, which includes the triad made up of Being, Nothing and Becoming, is something indeterminate and without any content. The content – for when we actually think in our daily lives we think in terms of contents – must come from somewhere else. But there is no other place outside of the Universe of Being, Nothing and Becoming. Outside of the Universe there is nothing that could function as limit or determination. Thus the limit must come from within the Universe itself.

Second step: From where could it come? Where is there a limit or delimitation within the Universe? Within the Universe there is negation. It is from negation, which is internal to the Universe, that comes the limit, delimitation or determination, for every negation is a determination. *Omnis determinatio est negatio*.

Third step: The Being that is further determined by negation is *Dasein*. This first form of negation transforms indeterminate Being into determinate Being, viz.,

into *Dasein*. But this determination is simple, i.e., something is determined by the negation of at least another something. And this another something is determined by the negation of the first something. One is determined by the other's negation. In other words: Determination is a game of relations of negation between two poles. A father is only father if he has a son, just as the son is only son if he has a father. The same applies to right and left, above and below, etc.

Fourth step: The question to be raised now is whether this does not lead to the perverse game of Bad Finitude and Bad Infinitude. For when we conceive something determinate (*Dasein* and Finitude), we inevitably end up in the endless repetition, in the *regressus ad infinitum*. What is the first determinant? If it is determinate, who or what determined it? And thus *ad infinitum*. The same occurs if we resort to a *progressus ad infinitum*. Both processes were unmasked in the previous chapter as being perverse. What then should we do?

Fifth step: Determination and Delimitation are forms of negation, of a negation that is internal to the Universe. From this there emerges *Dasein* with its simple determination, i.e., with its simple negation. When we focus on the father, viz. something determinate, we are always pointing also to the son. But we are focusing on the father, we are talking about the father, and not about the son. When we say that the father is tall, we are not saying anything about the son's height. In spite of the intimate relationship between father and son, which determines them as two poles, there is a big difference here. We focus on the father rather than the son, we speak of the father as something determinate without explicitly talking about the son. Thus the relationship between father and son has lost part of its priority and constitutive importance. The father as father, when conceived without making explicit reference to his relationship with his determinate son, is no longer mere *Dasein* (*Dasein*, Finitude, Infinitude). When the father is thus conceived as being in himself and for himself a Good Finitude and a Good Infinitude, he is conceived as a Being-for-itself. Therefore, Being-for-itself is just a simpler way of saying a Being with Good Finitude and Bad Infinitude. But after going through its otherness, the emphasis returns to the one from which we started. I.e., it is perfectly possible for the father to be an exemplary citizen even if the son is a scoundrel and vice-versa. It is as if the father has detached himself from the relationship between father and son (*aufheben* in the sense of overcoming) and is now considered by himself, in himself, as something that is a Being-for-itself.

1 1 3 A a Thesis: *Dasein* and Being-for itself

Being-for-itself, as discussed above, is *Dasein* with Good Finitude and Good Infinitude conceived again as something positive: it is the negation of negation. Through the first negation determinate *Dasein* emerges from indeterminate Being, and determinate *Dasein* proves to be of Good Finitude and Good Infinitude. Through the second negation, i.e., the negation of negation, we leave the multiplicity of poles that delimit and determine each other in *Dasein* and return on a higher level to the unity that is Being-for-itself. Thus every Being-for-itself is a Being for One, a being for a One.

But when we say this we return on a higher level to the problem which we already know of a determination that can only be made through negation. This is where thesis, antithesis and synthesis emerge from.

The thesis states: everything that was presupposed and must be repositied is a being for itself. – The thesis is false because being-for-itself, when posited by itself only in itself, is no longer something determinate, as it should be; in order to be determinate it should have an explicit negation of something other that would

determine it and distinguish it from everything else. What is the negation that determines Being-for-itself? What is the relationship of negation?

1 1 3 A b Antithesis: Being-for-One

Being-for-itself is not treated as a relationship between two different poles (e.g., father and son), but as a unity turned towards something that is simply one, Being-for-itself is first of all a Being-for-One.

The antithesis states: everything that was presupposed and must be repositated is Being-for-one. The antithesis is false because it does not explicitly state which one this one is. Of what one are we talking here? Hegel himself already asks in the table of contents: *Was für eines?* This "one" is indeterminate, vague and devoid of content; thus it cannot be the constitutive pole of a negative relationship of determination. Since there is no further determination of the One, it cannot determine what is the Being-for-itself that we are dealing with.

1 1 3 A c Synthesis: The One

In the dialectical back and forth between determination of one by the other (*Dasein*) and determination of Being-for-itself the One, the indeterminate and void One, reappears as an antithetical category. Insofar as this One is indeterminate, it constitutes only a false antithesis. But there is a One that is not indeterminate and void: viz., the One that is said and expressed in the "self". The one of the "self" contained in the category Being-for-itself is a One that is determinate and full of content. It is determined through the negation of the indeterminate One. The antithetical One is indeterminate because it does not point to anything. The synthetic One points to the "self" of the Being-for-itself.

Thus the determinate One is the dialectical synthesis in which *Dasein*, the Being-for-itself-as-such and the Being-for-One (the indeterminate one) are annulled and preserved. As we introduce the synthesis of the Being-for-itself that is a Being-for-the-determinate-One, which "is" the "self" of "being-for-itself", the relationship that determines the Being-for-itself with the One is a negation in the negation: it negates the indetermination of the indeterminate One of the antithesis and constitutes itself, by being turned to the "self" of itself, as a reflex and positive relationship. The One, conceived in this manner, is the dialectical abbreviation of the Being that, by going through the determinations of *Dasein* and of Being-for-itself-as-such, overcomes the differences and preserves and expresses the unity of the Universe.

All mediations – a typically Hegelian phrase – that we have made so far repeat and make explicit categories that we had already introduced and discussed in the comments in Formal Logic. For this reason it would be redundant to repeat here the formalizations made previously. The verbal explanation and the didactic reconstruction undertaken above should suffice. Therefore, let's move to 1 1 3 B, The One and the Many, for here we can axiomatize elements that have not been discussed with the necessary thoroughness.

1 1 3 B The One and the Many

8

The One is constructed by a double determination, viz., by a negation and by the negation of this negation. The first negation constitutes the One as something different from the Other. The second negation – the negation of the negation – states

that this something, although different from the Other, is flexed upon itself and, in this reflection upon itself, is much more than the mere negation of the Other. It is itself, it is the One.

1 1 3 B a The One in itself

The One, being determined by a double negation, contains the relationship with the Other as an overcome and preserved relationship. The One became itself again; its being is not *Dasein*, but neither is it the mere relationship with the Other. It contains (preserves) all these categories, but it also overcomes them (*aufheben*). It overcomes them because it leaves them behind, because it no longer makes them explicit, because it says more than they do. It preserves them because everything positive that they have is contained (*aufgehoben*) in it, in the One. The One insofar as it is abstracted from the history of its constitution, from the contents that were sublated (*aufgehoben*) is thus merely a One, an indeterminate and void One. In this sense the One becomes again a Nothing, viz., a complete void.

The thesis states: everything that was presupposed and must be repositied is the One. – This thesis is obviously false because it considers the Universe as something completely void and ignores the manifoldness and richness of things that inhabit it.

1 1 3 B b The One and the Void

The One is the abstract relationship of negation with itself: negation of negation. When we forget *Dasein*, the limit, determination by the Other – which were sublated, *overcome* – what was preserved as the content of the One is a complete Void. – The One conceived in this way is an atom, an atom with no relation to other atoms, an atom that is not in a field of forces. Thus it is an atom that does not say anything and is the pure Void in the solipsism of its absolute oneness.

The antithesis states: everything that was presupposed and must be repositied is the One and the Void. – This antithesis is false, since we are in a Universe full of manifoldness and variegated entities, including ourselves, who are making this Logic.

1 1 3 B c Many Ones

The transition from the unique and singular One to the many Ones is logically made because this One is in Becoming from its very beginning. The One that is in becoming is the One that demands another One, which in turn demands another One, and so forth.

The synthesis states: everything that was presupposed and must be repositied is “many Ones”, viz., the replication of the one that is reiterated *ad infinitum*. However, this infinitude, as previously demonstrated, becomes Good Infinitude and Good Finitude. It is something highly positive, it is the synthesis between the singular and unique One and the One that is replicated by constituting a numerical series in its Good Infinitude.

This complex and multifaceted relationship between the One and the Many will be formalized below. In order to avoid the repetitive turns made by Hegel from the positive to the negative meaning of the One and vice-versa, we introduce here the concepts of Good Unity and Good Multiplicity, which are constructed in similarity to the concepts of Good and Bad Infinitude. Thus the *progressus ad infinitum* in its bad sense is overcome.

G) Unity: The One and the Many

G1) Refutation of the thesis ‘ $\vdash x$ is of bad plurality’

G11) ADDITIONAL DEFINITIONS IN THE LT

Postulation

$GU_n x \Leftrightarrow \forall y x = y$	Good Unity [$GU_n x$: x is of good unity.]
$GPl x \Leftrightarrow \forall y x \neq y$	Good Plurality [$GPl x$: x is of good plurality.]
$BUn x \Leftrightarrow \neg GU_n x$	Bad Unity [$BUn x$: x is of bad unity.]
$BPl x \Leftrightarrow \neg GPl x$	Bad Plurality [$BPl x$: x is of bad plurality.]

G12) Conceptual Correspondences

Development

$GU_n x$	$\neg BUn x$	$\forall y x = y$	$\neg \Delta y x \neq y$	Synthesis (A)
$GPl x$	$\neg BPl x$	$\forall y x \neq y$	$\neg \Delta y x = y$	Synthesis (T)
$BUn x$	$\neg GU_n x$	$\neg \forall y x = y$	$\Delta y x \neq y$	Antithesis
$BPl x$	$\neg GPl x$	$\neg \forall y x \neq y$	$\Delta y x = y$	Thesis

G13) Theorem of the LT

$\vdash GPl x$ Good Plurality

Demonstration

1 $\vdash \forall y x \neq y$ Plurality [LT]
2 $\vdash GPl x$ 1 Good Plurality

G14) The thesis

Refutation

1 $\vdash GPl x$ Good Plurality [LT]
 $\left[\begin{array}{l} 2 \vdash BPl x \text{ Hypothetical Premise (Thesis)} \\ 3 \vdash \neg GPl x \text{ 1 Bad Plurality} \\ 4 \vdash GPl x \wedge \neg GPl x \text{ 1,3 Logical Product} \\ 5 \vdash f \text{ 4 Supplementation} \\ 6 f \text{ 5 Elimination of the Assertion} \end{array} \right.$
 7 $\vdash \neg (\neg GPl x)$ 1-6 Reduction to Absurdity

G2) Refutation of the antithesis ‘ $\vdash x$ is of bad unity’

G21) Theorem of the LT

$\vdash GU_n x$ Good Unity

Demonstration

1 $\vdash \forall y x = y$ Non-void [LBA]
2 $\vdash GU_n x$ 1 Good Unity

G22) The antithesis

Refutation

1 $\vdash GU_n x$ Good Unity [LT]
 $\left[\begin{array}{l} 2 \vdash BUn x \text{ Hypothetical Premise (Antithesis)} \\ 3 \vdash \neg GU_n x \text{ 2 Bad Unity} \\ 4 \vdash GU_n x \wedge \neg GU_n x \text{ 1,3 Logical Product} \\ 5 \vdash f \text{ 4 Supplementation} \\ 6 f \text{ 5 Elimination of the Assertion} \end{array} \right.$
 7 $\vdash \neg (\neg BUn x)$ 2-6 Reduction to Absurdity

G3) The synthesis of ‘ $\vdash x$ is of good plurality’ and ‘ $\vdash x$ is of good unity’

Thesis	$\vdash BPl x$	$\vdash x$ (existent or inexistent) is of bad plurality.
Antithesis	$\vdash BUn x$	$\vdash x$ (existent or inexistent) is of bad unity.
Synthesis (-) (T)	$\vdash \neg (\vdash BPl x)$	\vdash Non ($\vdash x$ (existent or inexistent) is of bad plurality).
Synthesis (-) (A)	$\vdash \neg (\vdash BUn x)$	\vdash Non ($\vdash x$ (existent or inexistent) is of bad unity).
Synthesis (+) (T)	$\vdash GPl x$	$\vdash x$ (existent or inexistent) is of good plurality.
Synthesis (+) (A)	$\vdash GUn x$	$\vdash x$ (existent or inexistent) is of good unity.

G4) Some apparently paradoxical developments of the synthesis

G411	$\vdash GPl x$ $\vdash x$ is of good plurality.	G421	$\vdash GUn x$ $\vdash x$ is of good unity.
G412	$\vdash \Delta x GPl x$ \vdash Everything is of good plurality.	G422	$\vdash \Delta x GUn x$ \vdash Everything is of good unity.
G413	$\vdash \nabla x GPl x$ \vdash Something is of good plurality.	G423	$\vdash \nabla x GUn x$ \vdash Something is of good unity.
G414	$\vdash \neg \nabla x BPl x$ \vdash Nothing is of bad plurality.	G424	$\vdash \neg \nabla x BUn x$ \vdash Nothing is of bad unity.
G415	$\vdash \neg \Delta x BPl x$ \vdash Not everything is of bad plurality.	G425	$\vdash \neg \Delta x BUn x$ \vdash Not everything is of bad unity.

In this formalization we can see that the issue of the One and the Many has the same structure that we find in the problem of the Good and Bad Finitude and the Good and Bad Infinitude. The philosophical kinship between both problems is conspicuous. – This exempts us from making many comments. If we compare what was discussed above about the One and the Many in natural language and the formalization we made, we can see that what is at stake is ultimately the same issue of Finitude und Infinitude, but worked out with somewhat richer concepts. This is precisely what Hegel intended and actually did.

1 1 3 C Repulsion and Attraction

Good Unity corresponds to Good Finitude and Good Multiplicity corresponds to Good Infinitude. In order to conceive the One and the Many correctly, they must be understood as a correlation of reciprocity in which the poles of the relationship are conciliated. If we do not do this, we inevitably end up either in the static one-



ness of Parmenides' sphere or in the disordered multiplicity of an atomism without the *clinamen*, which, by the way, nobody has ever proposed in philosophy.

However, in the system that Hegel and ourselves are proposing, the One and the Many (or Multiple) are always in Becoming. In other words, besides the mutually constitutive relationship between the Finite and the Infinite, between the One and the Multiple, one must conceive them as the One that is moving towards the Multiple and the Multiple that is moving towards the One. The One moving towards the Multiple is called Repulsion by Hegel. The Multiple moving towards the One is called Attraction. – Attraction is the co-belonging of all multiples, viz., of all numbers, to the One. Repulsion is the dynamic difference that makes the multiples different from the One, although all of them are, in themselves and each for itself, a One.

In Hegel attraction and repulsion constitute each other and are, for this reason, in balance. That is why the Universe, although it is always in Becoming, remains the Totality in Movement that it is, immobile and mobile at the same time. – But this is shown by Hegel in the habitual manner through thesis, antithesis and synthesis.

1 1 3 C a

The thesis states: everything that was presupposed and must be repositied is the exclusion of the One. – When it is stated that all the ones are just the reiteration of the One, all the many ones are conceived just as multiplicity, as the One that each one of them is in its oneness. This movement – which is a Becoming – is called Repulsion. Multiplicity is affirmed without unity. The thesis is false because if there were only Repulsion, the multifarious variety of beings would never constitute the unity of the Universe, leaving only many ones, each one of them singular and unrelated with the whole which is Totality in Movement. Furthermore, in the Universe there would exist only One and nothing else. There would be no thinking I that gathers thoughts, there would be no living beings in their systemic unity of various sub-systems, there would be no solar system nor the galaxies in their organization. This has already been refuted in the first chapter both in Logic and natural language.

1 1 3 C b

The antithesis states: everything that was presupposed and must be repositied is the One of Attraction. – The antithesis states precisely the opposite of the thesis: there is only the singular unity of the One. In other words, the Universe is only one single individual, the immobile and unchangeable sphere of Parmenides. There would exist neither movement nor the multiplicity of things. This has already been logically refuted when we talked, in the beginning, about the Universe that cannot be empty nor contain only one individual. There the negation of the multiplicity of things was refuted, since the Universe we live in does not allow us to affirm unity without multiplicity without falling into a performative contradiction.

1 1 3 C c

The synthesis states: everything that was presupposed and must be repositied is the relationship between repulsion and attraction. – The synthesis is true because it reestablishes the balance between the One and the Many also in their movement of Becoming. Attraction and Repulsion constitute each other



and must, therefore, be in balance. It is philosophically necessary that there is a balance between them, since one is defined by the other in a good and virtuous circularity.

In view of the arguments that have been presented, cosmologists would ask whether the Universe, according to this theory, can have begun in a Big Bang and whether it can finish in a Big Crush. Or else the Universe would have to be eternal, an eternal balance between Attraction and Repulsion. – When cosmologists talk about the Universe, they talk as representatives of a particular science that studies this Universe where we actually live. Most physicists – but not all of them – argue today for the theory of an initial Big Bang and many of them for the theory of a final Big Crush. Empirical evidence taken from our factual Universe seems to point in this direction. But from the philosophical point of view as above discussed there would have to be a perfect balance between attraction and repulsion. This would lead us to have a philosophical sympathy with the theses of those physicists who state (a) either that our World had a beginning and will have an end, but that the Universe as such, the big and total Universe, has neither beginning nor end and will remain eternally; (b) or that after each Big Crush there will be a new Big Bang.

In the logical formalization the issue of the One and the Many becomes the issue of Good Unity and Bad Unity, of Good Multiplicity and Bad Multiplicity. Here too we find the structural parallelism that pervades the problems of Good and Bad Finitude and Infinitude, of Good and Bad Unity and Multiplicity and appears in this case as Good and Bad Mobility and Immobility. Ultimately, the issue at stake here is just the One and the Many, but both of them in the movement of Becoming.

H) Mobility: The Mobile and the Immobile

H1) Refutation of the thesis ‘ $\vdash x$ is of bad immobility’

H11) ADDITIONAL DEFINITIONS IN THE LT

Postulation

$GMob\ x \Leftrightarrow \forall y\ Txy$	Good Mobility [<i>GMob</i> <i>x</i> : <i>x</i> is of good mobility.]
$GImb\ x \Leftrightarrow \forall y\ \neg Txy$	Good Immobility [<i>GImb</i> <i>x</i> : <i>x</i> is of good immobility.]
$BMob\ x \Leftrightarrow \neg GMob\ x$	Bad Mobility [<i>BMob</i> <i>x</i> : <i>x</i> is of bad mobility.]
$BImb\ x \Leftrightarrow \neg GImb\ x$	Bad Immobility [<i>BImb</i> <i>x</i> : <i>x</i> is of bad immobility.]

H12) Conceptual Correspondences

Development

$GMob\ x$	$\neg BMob\ x$	$\forall y\ Txy$	$\neg \Delta y\ \neg Txy$	Synthesis (A)
$GImb\ x$	$\neg BImb\ x$	$\forall y\ \neg Txy$	$\neg \Delta y\ Txy$	Synthesis (T)
$BMob\ x$	$\neg GMob\ x$	$\neg \forall y\ Txy$	$\Delta y\ \neg Txy$	Antithesis
$BImb\ x$	$\neg GImb\ x$	$\neg \forall y\ \neg Txy$	$\Delta y\ Txy$	Thesis

H13) Theorem of the LT

$\vdash GImb\ x$	Good Immobility
Demonstration	
1 $\vdash \forall y\ \neg Txy$	Non-transformation [LT]
2 $\vdash GImb\ x$	1 Good Immobility

H14) The thesis

Refutation

- 1 $\vdash GImb\ x$ Good Immobility [LT]
- | 2 $\vdash \neg BImb\ x$ Hypothetical Premise (**Thesis**)
- | 3 $\vdash \neg GImb\ x$ 1 Bad Immobility
- | 4 $\vdash GImb\ x \wedge \neg GImb\ x$
- | | 1,3 Logical Product
- | 5 $\vdash f$ 4 Supplementation
- | 6 f 5 Elimination of the Assertion
- 7 $\vdash \neg (\neg BImb\ x)$ 1-6 Reduction to Absurdity

H2) Refutation of the antithesis ‘ $\vdash x$ is of bad mobility’

H21) Theorem of the LT

$\vdash GMob\ x$ Good Mobility

Demonstration

- 1 $\vdash \forall y\ Txy$ Introduction of the Transformation [LBA]
- 2 $\vdash GMob\ x$ 1 Good Mobility

H22) The antithesis

Refutation

- 1 $\vdash GMob\ x$ Good Mobility [LT]
- | 2 $\vdash BMob\ x$ Hypothetical Premise (**Antithesis**)
- | 3 $\vdash \neg GMob\ x$ 2 Bad Mobility
- | 4 $\vdash GMob\ x \wedge \neg GMob\ x$
- | | 1,3 Logical Product
- | 5 $\vdash f$ 4 Supplementation
- | 6 f 5 Elimination of the Assertion
- 7 $\vdash \neg (\neg BMob\ x)$ 2-6 Reduction to Absurdity

H3) The synthesis of ‘ $\vdash x$ is of good immobility’ and ‘ $\vdash x$ is of good mobility’

Thesis	$\vdash BImb\ x$	$\vdash x$ (existent or inexistent) is of bad immobility.
Antithesis	$\vdash BMob\ x$	$\vdash x$ (existent or inexistent) is of bad mobility.
Synthesis (-) (T)	$\vdash \neg (\neg BImb\ x)$	\vdash Non ($\vdash x$ (existent or inexistent) is of bad immobility).
Synthesis (-) (A)	$\vdash \neg (\neg BMob\ x)$	\vdash Non ($\vdash x$ (existent or inexistent) is of bad mobility).
Synthesis (+) (T)	$\vdash GImb\ x$	$\vdash x$ (existent or inexistent) is of good immobility.
Synthesis (+) (A)	$\vdash GMob\ x$	$\vdash x$ (existent or inexistent) is of good mobility.

H4) Some apparently paradoxical developments of the synthesis

H411	$\vdash BGmb\ x$ $\vdash x$ is of good immobility.	H421	$\vdash GMob\ x$ $\vdash x$ is of good mobility.
H412	$\vdash \Delta x\ GImb\ x$ \vdash Everything is of good immobility.	H422	$\vdash \Delta x\ GMob\ x$ \vdash Everything is of good mobility.
H413	$\vdash \nabla x\ GImb\ x$ \vdash Something is of good immobility.	H423	$\vdash \nabla x\ GMob\ x$ \vdash Something is of good mobility.
H414	$\vdash \neg \nabla x\ BImb\ x$ \vdash Nothing is of bad immobility.	H424	$\vdash \neg \nabla x\ BMob\ x$ \vdash Nothing is of bad mobility.
H415	$\vdash \neg \Delta x\ BImb\ x$ \vdash Not everything is of bad immobility.	H425	$\vdash \neg \Delta x\ BMob\ x$ \vdash Not everything is of bad mobility.

1 2 Second part – Quantity

1 2 1 First chapter – Quantity

The difference between quantity and quality was explained previously. Quality is the first and immediate determination of Being; it is a Limit that limits and at the same time constitutes the Limited and the Unlimited, the Finite and the Infinite; it is a Being-for-itself that is for itself because it is also the Being-for-Other.

Since Being-for-itself is always also a Being-for-Other, the Limit between them is indifferent and flowing. It may be here just as it may as well be there. Determination is not fixed and, thus, completely determined, but flowing. It does limit, but one does not know exactly where, because it is something external to the categories that have been worked out so far. This indifference, this external character of the Limit that delimits Quality is called Quantity.

1 2 1 A Pure Quantity

The One repels and attracts itself at the same time. While it attracts itself and refers all other ones to itself, this Attraction is a form of continuity of the One that reiterates itself. This continuity is the immediate unity of the many ones: each One is outside of the other One, but all of them refer to the original One and originate in it. When it is said that each One is outside of the other, this already contains the characteristic of pure Quantity: *partes extra partes*. The emphasis is on the indetermination of the undifferentiated unity that becomes ambiguous here: do we mean to express attraction or repulsion? Continuity or discontinuity?

But in order to better understand the issue, we anticipate the antithesis discussed below: the One that attracts itself also repels itself. This repulsion in which each One repels the other ones and posits itself as different from them is a second essential characteristic of Quantity: besides being continuous (first part of the antithesis), it is also discrete (second part of the antithesis). Although all the ones refer to and originate in the main One, each one of them is a separate and discrete



One. Thus we have continuous Quantity and discrete Quantity, which constitute each other.

That is the reason why the arrow is still and moving at the same time. If we consider only the moment of continuity, the arrow enters an infinitely continuous space and will never come to a halt. If we consider only the moment of discontinuity, it does not move and will never reach the target: the still arrow. If we consider both aspects, the arrow crosses the space, which is continuous, but reaches the target, which is the moment of discontinuity. That is why Achilles wins the race, and the turtle does not.

The thesis states: everything that was presupposed and must be repositied is pure quantity. – This thesis is false because when Quantity is conceived as pure it does not yet make explicit the two elements that constitute it, viz., continuity and discontinuity. Since only continuity is expressed, the thesis is false and Zeno's and Kant's antinomies emerge, which are not able to conciliate continuity and discontinuity in Quantity.

1 2 1 B The continuous and discrete magnitude

When conceived as pure Quantity, Quantity is a false thesis because it is indeterminate, because the two elements that make it up are not made explicit, viz., continuity and discontinuity. The movement of unity of all ones in relation to the first One and the movement of dispersion in each One are different from all others.

The One that attracts itself also repels itself. This repulsion, in which each One repels the other ones and posits itself as different from them, is also an essential characteristic of Quantity; besides being continuous (first part of the antithesis), it is also discrete (second part of the antithesis). Although all the ones refer to and originate in the main One (continuity), each one of them is a separate and discrete One (discontinuity). Thus we have the continuous Magnitude (*Grösse*) and the discrete Magnitude, which constitute each other.

The antithesis states: everything that was presupposed and must be repositied is continuous and discrete magnitude. – The antithesis is false because it does not express the way in which the continuous and the discontinuous, which are initially separate and opposed, are conciliated and reunified. This is only done in the synthesis.

1 2 1 C Limitation of Quantity

In the thesis and antithesis above we saw that Quantity too is split into continuity and discontinuity. What unifies these two opposed elements? How are two opposed and mutually excluding elements conciliated and reunified?

It is the Limit that separates and opposes them to each other. There must be a Limit that makes possible the opposition between the continuous and the discontinuous. This Limitation of Quantity is the synthesis that is looked for. The Limitation between the two opposite elements is also what unifies and reconciles them: continuity and discontinuity constitute and define each other and they can be conceived without contradiction only when they are taken together.

The synthesis states: everything that was presupposed and must be repositied is a Limitation of Quantity. – The synthesis is true because it expresses the unity between the indetermination of the thesis, viz., pure Quantity, and the continuous and discontinuous Magnitude.



1 2 2 Second chapter – Quantum

Quantum is first of all Quantity with a Determination, i.e., with a Limit. When Quantum is conceived with the Limit as the only determination, it is simply the Number (1 2 2 A). When Quantum is understood as the Limit that delimits plurality, it is the extensive Quantum; it says how many unities it is delimiting. When Quantum is conceived as a unity that turns toward itself and fulfills itself as One, as Being-for-itself, it is the intensive Quantum, which indicates the quantitative degrees of this self-fulfillment (1 2 2 B). Quantum as the One and the Many in the realm of Quantity is also the quantitative Infinitude (1 2 2 C).

1 2 2 A Number

Quantity is here a Quantum and has a Limit, i.e., it is delimited both in its continuity and discontinuity. This Limit, however, is not located in a particular place; but wherever it is, it delimits Quantity and constitutes the One; in this sense it is the principle of continuity. Thus, this One is first of all the continuity of Quantity, which is one in and of itself. This One is, secondly, the principle of discontinuity, since the multiplicity of the many ones, which are different from each other, emerges from it. This One is, thirdly, the unity of continuity and discontinuity, because (a) it refers to itself in its unity, (b) it gives rise to the multiplicity of the many ones, which are, each one, different from the others, and (c) it gathers in itself both continuity and discontinuity.

When the three movements of the Quantum are posited and conceived simultaneously, we have the Number. The Number is the continuous and initial One, the series of many ones and the unity of this series. This leads us again to the problem of the good and bad Infinitude, since the numerical series is, on the one hand, infinite and can continue forever and, on the other, in its finitude it is determined and finite like this series. We will come back to this issue.

The thesis states: everything that was presupposed and must be repositied is Number. – This thesis is false, since it expresses the unity and multiplicity of the Quanta in an abstract manner. What is lacking here is the determination of the intensity or degree, which, as we saw, is part of the concept of Quantum.

1 2 2 B Extensive and intensive Quantum

Besides being continuous and discontinuous (see above), the Quantum can be extensive and intensive. When the Quantum is considered only in its numerical magnitude, it is extensive and asks only how many ones are within its limits; the answer here is always a determinate number of One, each one being different from the other. When the Quantum is considered only in its encompassing unity of many ones, it is intensive and indicates the degree. The degree does not tell how many ones are encompassed in the unity, but transforms the externality of the many ones into the internality of the degrees of intensity. – Both, however, constitute each other, because intensity too can be divided again and numbered when it is considered as discontinuous. In this way we obtain the numbering of the degrees of one single intensity, e.g., of heat. But both extension and intensity are in a process of Becoming, which gives rise to the problem of Infinitude.

The antithesis states: everything that was presupposed and must be repositied is an extensive and intensive quantum. – This antithesis is false too, because it left out the issue of Good and Bad Infinitude. Even when one talks about the Quantum in its extension and intensity, one must explicitly consider the issue of Infinitude.

1 2 2 C Quantitative Infinitude

The Quantum is continuous and discontinuous, extensive and intensive. In both dimensions there emerges the already known and previously discussed problem of the Good and Bad Finitude, of the Good and Bad Infinitude, for the Quantum, in any one of the two dimensions, contains a *progressus* and a *regressus ad infinitum*. One cannot adequately understand the Quantum if it is not brought, in its continuous and discontinuous and in its extensive and intensive dimensions, to the issue of Finitude and Infinitude. This problem, which we have already dealt with in the realm of Quality, returns here in the realm of Quantity and requires a solution. The answer is relatively simple: it is in principle the same that we have already proposed when we dealt with the issue of Good and Bad Infinitude in the realm of Quality. Good Finitude and Good Infinitude are the same, for they are the two sides of the same coin. – In this context Hegel discusses extensively the problems of the foundation of mathematics, differential calculus, Kant's antinomies and others. Against the background that we have delineated here, these problems can and must be dealt with; this has, in fact, already occurred and still occurs in mathematics (Leibniz, Frege, Russell, Peano, etc.). But they cannot be discussed here, because each one of them would require an entire book just for itself, but they do not have to be dealt with here, since ultimately the solutions are contained in what has been presented so far.

The synthesis states: everything that was presupposed and must be repositied is quantitative infinitude. – The synthesis is true and repositied the same answer already given to the problem of Infinitude. There is a Bad Finitude and a Good Finitude; there is a Bad Infinitude and a Good Infinitude. Good Finitude is the same as Good Infinitude; both of them constitute each other.

1 2 3 Third chapter – The Quantitative Relation

The Quantum as of Good Finitude and Good Infinitude is the unity of both elements, quantity and quality. This unity is called relation or ratio, as the ancient ones used to say.

Direct ratio exists if, with the growth of one element, the other element grows equally. A philosophically important example of direct and indirect ratio is found in the theological discussions about God's transcendence and immanence. God's transcendence and immanence are in a direct ratio (or relation) if, with the growth of one of them, the other one grows equally. Thus, the more transcendent God is, the more immanent God must be. If the kingdom of God is still to come (transcendence), then it is already here among us (immanence). In indirect ratio the opposite applies; the growth of one part implies the decrease of the other one. Thus, the more transcendent one conceives God to be, the less present (immanent) God is in this world.

Direct ratio is the thesis, indirect ratio is the antithesis and the relation of potency (*Potenzialverhältnis*) is the synthesis, according to Hegel. Below we will see what this means.

1 2 3 A Direct relation

Direct relation is the mere affirmative relation between the movement of two Quanta: when one of them grows, the other one grows too. The relation is immediate, direct and equal, always only in its positiveness.

The thesis states: everything that was presupposed and must be repositied is a direct relation. – This thesis is false because it is one-sided, because it expresses only the positiveness, but not the negativeness, because it does not express the variety of relations that exists in the world of ideas and things.

1 2 3 B Indirect relation

Indirect relation negates direct relation and affirms the opposite. In the movement between two Quanta, when one of them grows, the other one decreases. This relation is an indirect one, but it is mediated by negation and presents itself only in its negativeness.

The antithesis states: everything that was presupposed and must be repositied is an indirect relation. – This antithesis is false because it only expresses negativeness. It does not take into account the many direct relations that exist in the world of ideas and things.

1 2 3 C The relation of potency

Traditional interpreters of Hegel, led by the word “potency” (*Potenz*), look here for a mathematical relation similar to the potentiation that we know since the Greeks. Two in the second potency is four, four in the second potency is sixteen, and so forth. According to them, potency here means exactly the same as in arithmetic. But this interpretation is superficial and mistaken.

Hegel defines potency in this way: *Potenz ist eine Menge von Einheiten, deren jede diese Menge selbst ist* (Potency is a set of unities of which each one is this same set). The most common interpretation says that potency is a set of unities and that each one of these unities is the same. But the same as what? Is each unity the same as each one of the other unities? Or is each unity itself the same as the set that it constitutes? In the first case we have a banality: in the calculation of a second potency, one part is multiplied twice by itself; two in the second potency is four. In the second interpretation something completely new and apparently counter-intuitive appears: each unity, regardless of how many times it is potentiated, is always the same as the set of all unities. In mathematics this seems to be wrong. In Set Theory a class is only infinite if it is bijectable with a subclass of its own. In this case the whole is not larger than one of its (own) parts. In *Teoría axiomática de conjuntos* (2nd ed., Barcelona, Ariel, 1980, p. 115f.) Jesús Mosterín tells this delightful story: “Let us assume that a hotel with a finite number of rooms is completely occupied. If new customers arrive, there will be no place for them and they will not get a room. Let us now assume that the hotel has an infinity of rooms, as many as the natural numbers. If new customers arrive (even an infinite number of new customers), it will always be possible to receive them, even if the hotel is completely occupied. It will suffice, for instance, to invite the customers who already occupy it to move to the room with the double number of their present room. Thus, the guests of room 1 move to room 2, those of room 2 to 4, those of 3 to 6, ... those of n to $2n$. After that, all of them will continue having a room (with even numbers), but an infinity of rooms (those with uneven numbers) will be free. This extraordinary property of hotels with infinite rooms is the property that all infinite classes (and only they) have of being bijectable with a subclass of their own. This property was used by Dedekind to define infinitude.” But here we are dealing with philosophy, rather than mathematics. What Hegel and we mean is perhaps much more profound and difficult, for it would apply even to finite classes: each part, even when it is potentiated, is the same as the

whole of which it is a part. The part continues to be a part, but besides that it is always the whole of which it is a part.

In this second interpretation we have a far-reaching philosophical proposition, viz., the relation between part and whole. The part, however much it is a part, is always also the whole. This relation is not the "potentiation" from mathematics, but something much more encompassing and much higher. This is the most intimate relation between part and whole.

The positive and the negative relation, which in the beginning are opposed and exclude each other, are now conciliated and find their synthesis. Increase and decrease are no longer opposites, because we are no longer dealing with the relation between two parts, but with the relation between part and whole. The part may perfectly well increase or decrease without the whole having to increase or decrease. The really philosophical quantitative relation is the one that, leaving mathematics behind it, deals with the relation between the part and the Universe, between the part and Totality in movement.

The synthesis states: everything that is presupposed and must be repositied is a relation of potency. – The synthesis is correct because it conciliates and unifies the direct and indirect relation. Direct and indirect ratio are the same here because they are unified on a level higher than the level of mathematics, on the level of the philosophy of the whole and its parts.

This means that each being, by being a Quantum, is both of Good Finitude and Good Infinitude. Quantity and quality begin to be conciliated here.

I) Quality and Quantity

I1) Refutation of the thesis '¬ x is of bad quality'

I11) Additional Definitions in the LT

Postulation

$BQl x \Leftrightarrow \forall y \neg Txy \wedge \forall y x \neq y$	Good Quality [GQl x: x is of good quality.]
$GQt x \Leftrightarrow \forall y Txy$	Good Quantity [GQt x: x is of good quantity.]
$BQl x \Leftrightarrow \neg GQl x$	Bad Quality [BQl x: x is of bad quality.]
$BQt x \Leftrightarrow \neg GQt x$	Bad Quantity [BQt x: x is of bad quantity.]

I12) Theorem of the LT

$\vdash GQl x \Leftrightarrow (\forall y \neg Txy \wedge \forall y x \neq y \wedge \forall y \neg Lxy \wedge \forall y \neg Txy)$
Good Quality

Demonstration

- 1 $GQl x \Leftrightarrow \forall y \neg Txy \wedge \forall y x \neq y$
Good Quality
- 2 $\vdash GQl x \Leftrightarrow (\forall y \neg Txy \wedge \forall y x \neq y)$
1 Elimination of the Definition
- 3 $\vdash p \leftrightarrow p$ Identity
- 4 $\vdash (\forall y \neg Txy \wedge \forall y x \neq y) \leftrightarrow (\forall y \neg Txy \wedge \forall y x \neq y)$
3 Replacement of p by $\forall y \neg Txy \wedge \forall y x \neq y$
- 5 $\vdash \forall y \neg Lxy$ Active Limitation
- 6 $\vdash \forall y \neg Lxy \leftrightarrow v$ 5 Affirmation by Biconditional
- 7 $\vdash (\forall y \neg Txy \wedge \forall y x \neq y) \leftrightarrow ((\forall y \neg Txy \wedge \forall y x \neq y) \wedge v)$
4 Neutrality
- 8 $\vdash (\forall y \neg Txy \wedge \forall y x \neq y) \leftrightarrow$
 $\leftrightarrow ((\forall y \neg Txy \wedge \forall y x \neq y) \wedge \forall y \neg Lxy)$
6,7 Exchange

- 9 $\vdash (\forall y \neg Txy \wedge \forall y x \neq y) \leftrightarrow$
 $\leftrightarrow (\forall y \neg Txy \wedge \forall y x \neq y \wedge \forall y \neg Lxy)$
 8 Superfluous Parentheses
- 10 $\vdash \forall y \neg Fxy$ Passive Limitation
- 11 $\vdash \forall y \neg Fxy \leftrightarrow v$ 10 Affirmation by Biconditional
- 12 $\vdash (\forall y \neg Txy \wedge \forall y x \neq y) \leftrightarrow$
 $\leftrightarrow ((\forall y \neg Txy \wedge \forall y x \neq y \wedge \forall y \neg Lxy) \wedge v)$
 9 Neutrality
- 13 $\vdash (\forall y \neg Txy \wedge \forall y x \neq y) \leftrightarrow$
 $\leftrightarrow ((\forall y \neg Txy \wedge \forall y x \neq y \wedge \forall y \neg Lxy) \wedge \forall y \neg Fxy)$
 11,12 Exchange
- 14 $\vdash (\forall y \neg Txy \wedge \forall y x \neq y) \leftrightarrow$
 $\leftrightarrow (\forall y \neg Txy \wedge \forall y x \neq y \wedge \forall y \neg Lxy \wedge \forall y \neg Fxy)$
 13 Superfluous Parentheses
- 15 $\vdash GQl x \leftrightarrow (\forall y \neg Txy \wedge \forall y x \neq y \wedge \forall y \neg Lxy \wedge \forall y \neg Fxy)$
 2,14 Hypothetical Syllogism

I13) Theorem of the LT

- $\vdash GQt x \leftrightarrow (\forall y Txy \wedge \forall y x = y \wedge \forall y Lxy \wedge \forall y Fxy)$
 Good Quantity

Demonstration

- 1 $GQt x \Leftrightarrow \forall y Txy$ Good Quantity
- 2 $\vdash GQt x \leftrightarrow \forall y Txy$
 1 Elimination of the Definition
- 3 $\vdash p \leftrightarrow p$ Identity
- 4 $\vdash \forall y Txy \leftrightarrow \forall y Txy$
 3 Replacement of p by $\forall y Txy$
- 5 $\vdash \forall y x = y$ Non-void
- 6 $\vdash \forall y x = y \leftrightarrow v$ 5 Affirmation by Biconditional
- 7 $\vdash \forall y Txy \leftrightarrow (\forall y Txy \wedge v)$
 4 Neutrality
- 8 $\vdash \forall y Txy \leftrightarrow (\forall y Txy \wedge \forall y x = y)$
 6,7 Exchange
- 9 $\vdash \forall y Lxy$ Active Limitation
- 10 $\vdash \forall y Lxy \leftrightarrow v$ 9 Affirmation by Biconditional
- 11 $\vdash \forall y Txy \leftrightarrow ((\forall y Txy \wedge \forall y x = y) \wedge v)$
 8 Neutrality
- 12 $\vdash \forall y Txy \leftrightarrow ((\forall y Txy \wedge \forall y x = y) \wedge \forall y Lxy)$
 10,11 Exchange
- 13 $\vdash \forall y Txy \leftrightarrow (\forall y Txy \wedge \forall y x = y \wedge \forall y Lxy)$
 12 Superfluous Parentheses
- 14 $\vdash \forall y Fxy$ Passive Limitation
- 15 $\vdash \forall y Fxy \leftrightarrow v$ 14 Affirmation by Biconditional
- 16 $\vdash \forall y Txy \leftrightarrow ((\forall y Txy \wedge \forall y x = y \wedge \forall y Lxy) \wedge v)$
 13 Neutrality
- 17 $\vdash \forall y Txy \leftrightarrow ((\forall y Txy \wedge \forall y x = y \wedge \forall y Lxy) \wedge \forall y Fxy)$
 15,16 Exchange
- 18 $\vdash \forall y Txy \leftrightarrow (\forall y Txy \wedge \forall y x = y \wedge \forall y Lxy \wedge \forall y Fxy)$
 17 Superfluous Parentheses
- 19 $\vdash GQt x \leftrightarrow (\forall y Txy \wedge \forall y x = y \wedge \forall y Lxy \wedge \forall y Fxy)$
 2,18 Hypothetical Syllogism

I14) Theorem of the LT

- $\vdash BQl x \leftrightarrow (\Delta y Txy \vee \Delta y x = y \vee \Delta y Lxy \vee \Delta y Fxy)$
 Bad Quality

Demonstration

- 1 $\vdash GQl x \leftrightarrow (\forall y \neg Txy \wedge \forall y x \neq y \wedge \forall y \neg Lxy \wedge \forall y \neg Fxy)$
Good Quality
- 2 $\vdash \neg GQl x \leftrightarrow \neg (\forall y \neg Txy \wedge \forall y x \neq y \wedge \forall y \neg Lxy \wedge \forall y \neg Fxy)$
1 Inversion
- 3 $\vdash \neg GQl x \leftrightarrow$
 $\leftrightarrow (\neg \forall y \neg Txy \vee \neg \forall y x \neq y \vee \neg \forall y \neg Lxy \vee \neg \forall y \neg Fxy)$
2 Superfluous Parentheses and De Morgan
(iteratively repeated)
- 4 $\vdash \neg GQl x \leftrightarrow$
 $\leftrightarrow (\neg \forall y \neg Txy \vee \neg \forall y x = y \vee \neg \forall y \neg Lxy \vee \neg \forall y \neg Fxy)$
3 Otherness
- 5 $\vdash \neg GQl x \leftrightarrow (\Delta y Txy \vee \Delta y x = y \vee \Delta y Lxy \vee \Delta y Fxy)$
4 Resolution of the Bound Assertion (4 times)
- 6 $BQl x \approx \neg GQl x$
Bad Quality
- 7 $\vdash BQl x \leftrightarrow \neg GQl x$
6 Elimination of the Definition
- 8 $\vdash BQl x \leftrightarrow (\Delta y Txy \vee \Delta y x = y \vee \Delta y Lxy \vee \Delta y Fxy)$
7,5 Hypothetical Syllogism

I15) Theorem of the LT

- $\vdash BQt x \leftrightarrow (\neg \forall y Txy \vee \neg \forall y x = y \vee \neg \forall y Lxy \vee \neg \forall y Fxy)$
Bad Quantity

Demonstration

- 1 $\vdash GQt x \leftrightarrow (\forall y Txy \wedge \forall y x = y \wedge \forall y Lxy \wedge \forall y Fxy)$
Good Quantity
- 2 $\vdash \neg GQt x \leftrightarrow \neg (\forall y Txy \wedge \forall y x = y \wedge \forall y Lxy \wedge \forall y Fxy)$
1 Inversion
- 3 $\vdash \neg BQt x \leftrightarrow (\neg \forall y Txy \wedge \neg \forall y x = y \wedge \neg \forall y Lxy \wedge \neg \forall y Fxy)$
2 Superfluous Parentheses and De Morgan
(iteratively repeated)
- 4 $BQt x \approx \neg GQt x$ Bad Quantity
- 5 $\vdash BQt x \leftrightarrow \neg GQt x$
4 Elimination of the Definition
- 6 $\vdash BQt x \leftrightarrow (\neg \forall y Txy \wedge \neg \forall y x = y \wedge \neg \forall y Lxy \wedge \neg \forall y Fxy)$
5,3 Hypothetical Syllogism

I16) Conceptual correspondences

Development

1	$GQt x$	$\neg BQt x$	$\forall y Txy$	Synthesis (A)
2	$GQl x$	$\neg BQl x$	$\forall y \neg Txy \wedge \forall y x \neq y$	Synthesis (T)
3	$BQt x$	$\neg GQt x$	$\neg \forall y Txy$	Antithesis
4	$BQl x$	$\neg GQl x$	$\Delta y Txy \vee \Delta y x = y$	Thesis

1	$\forall y \neg Txy \wedge \forall y x \neq y \wedge \forall y \neg Lxy \wedge \forall y \neg Fxy$	Synthesis (A)
2	$\forall y Txy \wedge \forall y x = y \wedge \forall y Lxy \wedge \forall y Fxy$	Synthesis (T)
3	$\Delta y Txy \vee \Delta y x = y \vee \Delta y Lxy \vee \Delta y Fxy$	Antithesis
4	$\neg \forall y Txy \vee \neg \forall y x = y \vee \neg \forall y Lxy \vee \neg \forall y Fxy$	Thesis

I17) Theorem of the LT

- $\vdash GQl x$ Good Quality
- Demonstration
- 1 $GQl x \Leftrightarrow \forall y \neg Txy \wedge \forall y x \neq y$ Good Quality
- 2 $\forall y \neg Txy$ Transformation
- 3 $\forall y x \neq y$ Plurality
- 4 $\forall y \neg Txy \wedge \forall y x \neq y$ 2,3 Logical Product
- 5 $GQl x$ 1,4 Separation

I18) The thesis

- Refutation
- 1 $GQl x$ Good Quality
- ┌ 2 $BQl x$ Hypothetical Premise (Thesis)
- ├ 3 $\neg GQl x$ 2 Bad Quality
- ├ 4 $GQl x \wedge \neg GQl x$ 1,3 Logical Product
- ├ 5 f 4 Supplementation
- └ 6 f 5 Elimination of the Assertion
- 7 $\neg (K BQl x)$ 2-6 Reduction to Absurdity

I2) Refutation of the antithesis ‘ $\vdash x$ is of bad quantity’

I21) Theorem of the LT

- $\vdash GQt x$ Good Quantity
- Demonstration
- 1 $GQt x \Leftrightarrow \forall y Txy$ Good Quantity
- 2 $GQt x \leftrightarrow \forall y Txy$ 1 Elimination of the Definition
- 3 $\forall y Txy$ Introduction of the Transformation
- 4 $GQt x$ 2,3 Separation

I22) The antithesis

- Refutation
- 1 $GQt x$ Good Quantity
- ┌ 2 $BQt x$ Hypothetical Premise (Thesis)
- ├ 3 $\neg GQt x$ 2 Bad Quantity
- ├ 4 $GQt x \wedge \neg GQt x$ 1,3 Logical Product
- ├ 5 f 4 Supplementation
- └ 6 f 5 Elimination of the Assertion
- 7 $\neg (\vdash BQt x)$ 2-6 Reduction to Absurdity

I3) The synthesis of ‘ $\vdash x$ is of good quality’ and ‘ $\vdash x$ is of good quantity’

Thesis	$\vdash BQl x$	$\vdash x$ (existent or inexistent) is of bad quality.
Antithesis	$\vdash BQt x$	$\vdash x$ (existent or inexistent) is of bad quantity.
Synthesis (-) (T)	$\vdash \neg (\vdash BQl x)$	\vdash Non ($\vdash x$ (existent or inexistent) is of bad quality).
Synthesis (-) (A)	$\vdash \neg (\vdash BQt x)$	\vdash Non ($\vdash x$ (existent or inexistent) is of bad quantity).
Synthesis (+) (T)	$\vdash GQl x$	$\vdash x$ (existent or inexistent) is of good quality.
Synthesis (+) (A)	$\vdash GQt x$	$\vdash x$ (existent or inexistent) is of good quantity.

14) Some apparently paradoxical developments of the synthesis

I411	$\vdash GQl x$ $\vdash x$ is of good quality.	I421	$\vdash GQt x$ $\vdash x$ is of good quantity.
I412	$\vdash \Delta x GQl x$ \vdash Everything is of good quality.	I422	$\vdash \Delta x GQt x$ \vdash Everything is of good quantity.
I413	$\vdash \nabla x GQl x$ \vdash Something is of good quality.	I423	$\vdash \nabla x GQt x$ \vdash Something is of good quantity.
I414	$\vdash \neg \nabla x BQl x$ \vdash Nothing is of bad quality.	I424	$\vdash \neg \nabla x BQt x$ \vdash Nothing is of bad quantity.
I415	$\vdash \neg \Delta x BQl x$ \vdash Not everything is of bad quality.	I425	$\vdash \neg \Delta x BQt x$ \vdash Not everything is of bad quantity.

1 3 Third part – Measure

The dialectical development of the categories on Quantity showed that there is no Quantity without Quality, just as there is no Quality without Quantity. In the second part we saw that all categories that refer to Quantity always contain a qualitative element. Without this qualitative element Quantity would be a mere Non-being, a Nothing, void, undifferentiated, deprived of any determination. For Quantity to be Quantity, it must be the determinate negation of a Quality, i.e., Quantity is always the Quality that goes outside of itself, almost loses its sameness but never completely loses the identity with itself. Quantity is Quality outside of itself, an Internality that got lost in Externality. The classical phrase *partes extra partes* expresses both aspects. The word *partes* points to Quality; the word *extra* means negation, Non-identity, Externality, which is Quantity. For this reason in the treatment of Quantity we had to work out the concept of Quantum, a determinate quality that deals with the problems of Good and Bad Infinitude, of Good and Bad Finitude. The very concept of Quantum turns toward itself and discovers itself as Quantitative Relation. Quantitative Relation is just the Quantum that is conscious of itself, that discovers itself as self-relative and self-referential.

This would lead us again to the problem of Bad Infinitude, for a relation that is flexed upon itself, that is self-relation, loses, as it seems, all and any determination. One does not know where it begins and where it ends; one does not know how it delimits itself. Thus, with the category of Quantitative Relation we are back in the old problem of the vicious circle, which, besides being vicious for presupposing itself, is empty.

This is the problem that is discussed in the third and last part of the first book under the title of Measure. Measuring is a very special form of determining and specifying. One takes a determinate quantity, for instance a meter, and applies it to another determinate quantity, for instance a lasso; the successive application of the same meter – a determinate quantity – to the length of the lasso will determine how many meters this lasso has. The working lasso of an adult “gaucho” must have, for traditional and functional reasons, 26.4 meters. – We may take another determinate quantity, the foot. By using the length of the foot as a measuring

standard and by applying this standard to other determinate quantities, we can say that a house, for instance, is 27 feet in front by 90 feet long to the rear. – In both examples mentioned we took a determinate quantity, i.e., a Specific Quantity, but besides that, in a flexion of quantitative ratio itself, we are considering this determinate Quantum as a standard of measure and began to determine other determinate quantities, other Quanta, on the basis of how many times they contain the adopted standard. Measure is a further determination of the specific Quantum and Quantitative Relation because it establishes a *sui generis* kind of self-flexion and self-determination. This is the topic of this third part, which presents itself as a synthesis between Quality (first part) and Quantity (second part). The synthesis between both consists of an Identity that is flexed upon itself, applies itself as a standard to itself and is thus determined as self-flexion, i.e., as self-determination. This is the last category in the Logic of Being (first book) and makes the transition to the Logic of Essence (second book). Measure is a more sophisticated form of self-reference, of self-flexion, of reflexion. The Logic of Essence deals exclusively with these circular structures that, being vicious in the beginning, prove to be virtuous at a later point.

1 3 1 First chapter – Specific Quantity

1 3 1 A Specific Quantum

Measure is a Specific Quantum that is taken as standard and, by being applied to other Specific Quanta, tells how many times one is contained in the other: the first in the latter ones (How many times do the Quanta to be measured contain the standard Quantum?) or the latter in the first one (How many Quanta to be measured are contained how many times in the standard Quantum?). I.e., ,how many feet is the front of the house? Or how many beans are on a stem?

The feature that stands out in this context is the empirical and arbitrary character of the standard of measure. From an *a priori* philosophical point of view, it is very clear what a Measure is: you take a determinate Quantum as standard and apply it on other determinate quantities. But his choice of a standard is empirical and arbitrary. Are we going to use as a standard of measure the inch, the hand, the foot, the yard, the metal bar kept in a well in Paris (the meter) or what? Even if – which is not the case – the foot were a measuring standard, the feet are empirical, *a posteriori*, and do not have precisely the same size. Which one then should we choose as a standard? Here the arbitrariness of the standard of measuring appears with total clarity. Measure and standard of measure are relational concepts: measuring consists of the self-application of the determinate Quantum to other determinate Quanta. They are *a priori* concepts. But the choice of the standard to be used, whatever it is, is something empirical, arbitrary, fruit of the tradition or social convention. The Specific Quantity that is the thesis here is arbitrary because it is indeterminate in itself; although it is called Specific Quantum, one does not know where it begins and ends. It continues to be indeterminate.

The thesis states: everything that was presupposed and must be reposited is the specific quantity. – The thesis is false because it is arbitrary, because it contains something outside of itself that tells where it begins and ends. But the categories of Logic in the Hegelian sense cannot be determined by something that is outside of the Universe, outside of the Totality in Movement.



1 3 1 B The Specifying Measure

For a determinate Quantum to become a standard of measure, an act of choice is necessary: an act that chooses among several possibilities (inch, hand, foot, yard etc.) a determinate Quantum as being specific and turns it into the standard of measure. The standard of measure in itself is something arbitrary and empirical; but once it has been chosen as a standard, it acquires a scientific function and becomes the Specifying Measure. If the foot is chosen as standard of measure, all members of that particular culture and all its sciences must measure the quantities by saying how many feet they have. The Quantum is no longer indeterminate; it became specific and concrete. But it still has its characteristic of arbitrariness. The Specifying Measure too, although it is a determinate Quantum, is something that receives its determination from outside itself. Such a concept cannot be the universal category of Logic, because it presupposes something outside of the Universe.

The antithesis states: everything that was presupposed and must be repositied is a specifying measure. – This antithesis is false too, since it contains a determining element that comes from outside of the Universe, from outside of the Totality in Movement.

1 3 1 C Being-for-itself in the Measure

To be understood philosophically, the Measure requires that the element that determines and specifies the Quantum must come from within the Universe, from within the Totality in Movement. For the measure to be able to be conceived as a philosophical category, the determination of the Specific Quantum and the Specifying Quantum has to come from within itself. In philosophy the Measure must be a Being-for-itself, i.e., the Measure must determine itself from within itself. The Measure must be something self-determined.

The synthesis states: everything that was presupposed and must be repositied is a Measure that is a Being-for-itself. – The synthesis, which is true, expresses the philosophical need that the Quantum determines itself as Measure, without arbitrariness and without merely empirical conventions. The determination must come from within the Totality in Movement, from within the Measure itself while the latter is conceived as a philosophical category.

1 3 2 Second chapter – The Real Measure

1 3 2 A The Relation of Independent Measures

In the triad of the previous chapter the Measure was determined as a relation that exists between determinate and thus concrete Quanta. In the Measure a quantitative relation between at least two quantitative and concrete bodies is established. This Measure, although arbitrary, is autonomous, i.e., it rules itself (Being-for-itself). Obviously these various independent and autonomous measures are not isolated, for they constitute a network of mutual relations: they are a Relation of Independent Measures.

Here Hegel discusses first the relation between two measures. Secondly, Measure is considered as a series of measure relations. Thirdly, he deals with the measures that are called *Wahlverwandtschaften* (elective affinities). Hegel, Goethe and various authors of German Romanticism, as they did not know the atomic model of Niels and Rutherford that was discovered only later on, thought that certain substances had a special attraction to other substances. In their view, some persons felt especially attracted to other persons, which originated love relationships based on



elective affinities (see Goethe's novel titled *Wahlverwandtschaften*). Today we know that the so-called affinities in physics and chemistry have their origin in the atomic structure and are ruled by laws that determine why an element or particle attracts or repels another one. There are cosmic forces of attraction and repulsion, such as electromagnetism, and others that seem only to attract (although not all physicists agree with this), such as the force of gravity. There is also the duplicity of particles, i.e., of the matter that we observe and of the black matter, of whose existence we are aware without knowing exactly what it is. In physics we are lacking the Great Unified Theory (GUT), which aims at unifying in a single theory the four big forces that we work with today. – From the merely philosophical point of view we cannot advance in this respect and have to wait for the results from physics.

The – false – thesis in this chapter on the Real Measure states that the Universe is composed of a Relation of Independent Measures.

The thesis states: everything that was presupposed and must be repositied is a relation of independent measures. – The falseness of the thesis is due to the fact that it only deals with determinate quantities, with the relation of specific Measures, with the series of relations of measure and with the Measures called elective affinities. What is lacking here are the knots that are formed between the relations of measure. What is lacking here is the unity that conciliates theses multiplicities.

1 3 2 B The knots between the relations of Measure

If the relations of Measure are not further specified and determined, they are dispersed in multiplicity. They are multiple, but do not express the unity that must keep them united among themselves and conciliated with themselves. Here the antithesis is constituted by the knots between the relations of Measure. Knot means, as the word itself expresses it, junction and thus unity. The thesis about the relation of independent measures is opposed by the antithesis about the knots between the relations of measure. The knots mean precisely the lacking unity.

The antithesis states: everything that was presupposed and must be repositied is a relation of knots between the relations of Measure. – This antithesis is false too. It is opposed to the thesis in the way unity is opposed to multiplicity. What is lacking now is the unity between unity and multiplicity. But how are we to constitute a unity between the One and the Many of Real Measures? Hegel's answer is a hard one and raises a problem that has to be dealt with: the Measureless.

1 3 2 C The Measureless

The Measureless is the attempt – the heroic attempt, as the Greeks say – to unify the relation of independent measures in their plurality with the knots between the relations of measure in their unity. The Measureless seems to be here the conciliation between the One and the Many, because both are fused and disappear in it.

The synthesis states: everything that was presupposed and must be repositied is a measureless. – This synthesis is true because and to the extent that it unifies the One and the Many in the relations of Measure. In this respect it is a synthesis here. In the Measureless the One comes to the fore and the Multiple almost disappears.

But this synthesis immediately leaves its agglutinative position and becomes again a thesis to be worked on, for the Measureless, precisely because it is without measure, has sublated (*aufheben*) the element of Multiplicity to such an extent that the element of unity has almost disappeared in the indifferentiation of something which, having no Measure (*Massloses*), loses almost all its determinations. This last



synthesis is so unifying (as it sublates multiplicity) that it has not kept in a sufficiently explicit manner the multiplicity that it contains. Thus it becomes again a thesis and leads to the next, third and last, chapter: The Becoming of Essence.

The Measureless is in fact a synthesis; we find it in the Greek heroes. But it is at the same time its perdition. For without Measure the Greek hero leaves Epos, is no longer a hero and falls in the inexorable snare of Tragedy.

1 3 3 Third chapter – The Becoming of the Essence

1 3 3 A Absolute Indifference

Being is abstract indifference, where there is yet no determination; pure Quantity acquires some determination when it is conceived as Quantum and particularly as Measure, because in Measure determination becomes more conspicuous. Nonetheless, the question that keeps returning and cannot be silenced is this: How can determination come from within the Universe, from within the Totality in Movement? Every time we think we can catch it, it escapes us again and comes from the outside. This also occurred in the case of the Measure, in the knots of the determinate measures. Thus we were led to another absurdity. If determination does not come from the outside, then the whole Universe, the whole Totality in Movement is a Measureless in which ultimately there is nothing really concrete and determinate. And where are we in our determination? Where are the multiple and variegated determinations of the Universe we live in?

The reasoning developed in the last triad, especially in the category of the Measureless, tells us that the Universe is itself absolute indifference to any determination. The Universe contains determinations, but it is none of them.

The thesis states: everything that was presupposed and must be repositied is absolute indifference. – This thesis is obviously false, for the Universe is in itself determinate, just as we are determinate in it. There is determination rather than absolute indifference. That is why the thesis is false.

1 3 3 B Indifference as the Inverse Relation of its Factors

So far we have looked for the determination of each category in a whole that is larger and richer than itself, in a whole of which it is a part. The determinations that specify each category of this Logic – which is an Ontology – receive their determination from a whole which, from the point of view of its content, is larger and richer than the categories. In this sense, the determination of each category comes to it from the outside. Not from the outside of the Universe, because there is nothing outside of the Universe. But from the outside of its set of already explicitly developed contents, viz., of already repositied contents (see the unnumbered chapter in the beginning on presupposing and subsequent repositing). The further explicit development of each category adds something new to it, a new conceptual richness, i.e., a new determination. Now it is no longer possible to postpone this question: If the determinations that further characterize each category come from this richness of contents that is still hidden in the totality in movement, how can we apprehend and reposit them in the system? All Metaphysics in its history has tried to solve this question and thus has created the most diverse binomials, i.e., games of opposites. But these games of opposites have usually been ill-constructed because the relation's opposites do not have the same weight and are not co-original; usually one of the



relation's opposites has more weight and functions as the starting point, which is wrongly considered as absolute (essence and appearance, substance and accident, form and matter etc.) Thus the history of Metaphysics became a history of errors, since one of the members of the binomial – of the game of opposites – was considered as the really real reality (*ontos on*) and the other element of the binomial was relegated to a shadowy existence or at most to a secondary and auxiliary moment. Thus it is assumed that the really real reality is expressed by essence rather than by appearance, by form rather than by matter, by the substance rather than by the accident etc., which is a big mistake, as we shall see in the Logic of Essence.

The antithesis states: everything that was presupposed and must be repositied is indifference as the inverse relation of its factors. – The antithesis is false because it merely reverses the poles of the relation that exists in the midst of the thetical Indifference. The determination of one pole by the other one is a relation that can be reversed. Why do we not say – what are the reasons for not saying – that the really real reality is the accident rather than the substance? Appearance rather than essence? Matter rather than form?

The absolute Indifference, which is the false thesis, is opposed by the determination through an inversion of factors for the further determination of the categories. This antithesis is false too, since the two poles of the relation are mutually constituted and must be conceived together, for otherwise we are thrown from one to the other in an endless game. The attempt to determine the categories by taking one pole of the relation as the main and dominant, viz., determinant one leads us to the problem of inversion: Why don't we start from the other pole and do the same? But there is a more serious and definitive argument: The relation can be correctly determined only by starting at the same time from the two poles that constitute it. Without this there is no relation that is originally relative, i.e., in which each pole really constitutes the other one. This is why the mere inversion of the poles proves to be a wrong antithesis.

1 3 3 C Transition to Essence

The history of philosophy is today the long and winding road on whose roadside lie the white skeletons of philosophical systems that have erred, gotten lost and died because they did not realize that in a philosophical system the two poles of the relations must be conceived and determined at the same time, one by the other, one constituting the other.

This gives rise immediately to the question whether this is not a vicious circle. Isn't that precisely the error of the *circulus vitiosus*?

Yes and no. The second book of Hegel's *Science of Logic*, which is perhaps the most critical work that has ever been written about the basic concepts of traditional philosophy, shows in a detailed way how the great systems erred precisely because they fell in this vicious circle. One by one, all the great systems are reviewed – without mentioning the authors' names – and unmasked as vicious circles. – But is there a way out? Is there a way to get out of the vicious circle that pervades all our tradition? Hegel shows that there is in fact a way out: it is necessary to transform the vicious circles into virtuous ones. And he does it brilliantly in most cases. In one case, however, in the dialectics of modalities, Hegel is at least ambiguous. In the development of the category of Absolute Necessity he is not clear, and the whole system takes on a character of necessitarianism that distorts it. From this point onwards, Hegel – as Spinoza did before him – moves almost always within

the parameters of systemic necessitarianism that renders free choice and thus true ethics impossible. – Precisely at this point we shall make a corrective interpretation of the system. We shall transform absolute necessity, which is considered by Spinoza and Hegel as a hard necessity that does not admit counterfactuals and applies to Logic, Mathematics and certain parts of Philosophy, into a soft necessity that allows counterfactuals and applies to the subatomic world, to the Theory of Evolution of living beings and societies and particularly to what we call Ethics and Law. The soft necessity is the major law of the system, it is the larger set within which lies the hard necessity that is smaller and less encompassing. Soft necessity will be the great law, the really universal law that applies to the whole Universe and rules Totality in Movement.

The synthesis states: everything that was presupposed and must be repositated is the transition to essence. – The synthesis is in fact the transformation of vicious circles into virtuous ones. This is the true synthesis that opens the doors for us to the second book of the *Science of Logic*.

J) Active Measure

Previous Note on the Measure, both Active (item J) and Passive (item K):

The relation of measure (both active and passive) is defined by the relation of identity. Thus all properties of the relation of identity apply to the relation of measure (both active and passive), such as exchange, reflexivity, symmetry, transitivity. Exchange would mean the following: if x measures y , then if x is P , then y is P . This means that what measures and what is measured have the same properties. In other words, “the form (the properties) of the recipient (of what measures) and the form of the received (of what is measured) are identical.”

J1) Refutation of the thesis ‘ $\vdash x$ measures everything’

J11) Additional Definition

Postulation

$$Mxy \Leftrightarrow x = y \qquad Mxy: x \text{ measures } y$$

J12) Theorem of the LT

$\vdash \forall y \neg Mxy$	Measure
Demonstration	
1 $\vdash \forall y x \neq y$	Plurality
2 $Mxy \Leftrightarrow x = y$	Measure
3 $\vdash Mxy \leftrightarrow x = y$	2 Elimination of the Definition
4 $\vdash \neg Mxy \leftrightarrow \neg x = y$	3 Inversion
5 $\vdash \neg Mxy \leftrightarrow x \neq y$	4 Otherness
6 $\vdash \Delta y (\neg Mxy \leftrightarrow x \neq y)$	5 Introduction of the Generalizing Assertion
7 $\vdash \forall y \neg Mxy \leftrightarrow \forall y x \neq y$	6 Distribution of the Bound Assertion
8 $\vdash \forall y \neg Mxy$	7,1 Separation

J13) The thesis

Refutation

1 $\vdash \forall y \neg Mxy$	Measure
┌ 2 $\vdash \Delta y Mxy$	Hypothetical Premise (Thesis)
└ 3 $\vdash \neg \forall y \neg Mxy$	2 Resolution of the Bound Assertion
└ 4 $\vdash \forall y \neg Mxy \wedge \neg \forall y \neg Mxy$	1,3 Logical Product
└ 5 $\vdash f$	4 Supplementation
└ 6 f	5 Elimination of the Assertion
7 $\vdash \neg (\vdash \Delta y Mxy)$	2-6 Reduction to Absurdity

J2) Refutation of the antithesis ‘ $\vdash x$ measures nothing’

J21) Theorem of the LT

$\vdash Mxx$ Measure

Demonstration

- 1 $Mxy \Leftrightarrow x = y$ Measure
- 2 $\vdash Mxy \leftrightarrow x = y$ 1 Elimination of the Definition
- 3 $\vdash Mxx \leftrightarrow x = x$ 2 Replacement of x by y
- 4 $\vdash x = x$ Reflexivity
- 5 $\vdash Mxx$ 3,4 Separation

J22) Theorem of the LT

$\vdash \forall y Mxy$ Measure

Demonstration

- 1 $\vdash \forall y x = y$ Non-void
- 2 $Mxy \Leftrightarrow x = y$ Measure
- 3 $\vdash Mxy \leftrightarrow x = y$ 2 Elimination of the Definition
- 4 $\vdash \Delta y (Mxy \leftrightarrow x = y)$ 3 Introduction of the Generalizing Assertion
- 5 $\vdash \forall y Mxy \leftrightarrow \forall y x = y$ 4 Distribution of the Bound Assertion
- 6 $\vdash \forall y Mxy$ 5,1 Separation

J23) The antithesis

Refutation

- 1 $\vdash \forall y Mxy$ Measure
- 2 $\vdash \neg \forall y Mxy$ Hypothetical Premise (**Antithesis**)
- 3 $\vdash \forall y Mxy \wedge \neg \forall y Mxy$
- 4 $\vdash f$ 1,2 Logical Product
- 5 f 3 Logical Product
- 6 $\vdash \neg (\vdash \forall y Mxy)$ 4 Elimination of the Assertion
- 7 $\vdash \neg (\vdash \neg \forall y Mxy)$ 2-5 Reduction to Absurdity

J3) The synthesis of ‘ $\vdash x$ measures something’ and ‘ $\vdash x$ does not measure something’

Thesis	$\vdash \Delta y Mxy$	$\vdash x$ (existent or inexistent) measures everything (existent or inexistent).
Antithesis	$\vdash \neg \forall y Mxy$	$\vdash x$ (existent or inexistent) measures nothing (existent or inexistent).
Synthesis (-) (T)	$\vdash \neg (\vdash \Delta y Mxy)$	\vdash Non ($\vdash x$ (existent or inexistent) measures everything (existent or inexistent).
Synthesis (-) (A)	$\vdash \neg (\vdash \neg \forall y Mxy)$	\vdash Non ($\vdash x$ (existent or inexistent) measures nothing (existent or inexistent).
Synthesis (+) (T)	$\vdash \forall y \neg Mxy$	$\vdash x$ (existent or inexistent) does not measure something (existent or inexistent).
Synthesis (+) (A)	$\vdash \forall y Mxy$	$\vdash x$ (existent or inexistent) measures something (existent or inexistent).

K14) The thesis

Refutation

- 1 $\vdash \nabla y \neg Wxy$ Measure
 - ┌ 2 $\vdash \Delta y Wxy$ Hypothetical Premise (**Thesis**)
 - ├ 3 $\vdash \neg \nabla y \neg Wxy$ 2 Resolution of the Bound Assertion
 - ├ 4 $\vdash \nabla y \neg Wxy \wedge \neg \nabla y \neg Wxy$
 - ├ 5 $\vdash f$ 1,3 Logical Product
 - ├ 6 f 4 Supplementation
 - └ 7 $\vdash \neg (\vdash \Delta y Wxy)$ 5 Elimination of the Assertion
- 2-6 Reduction to Absurdity

K2) Refutation of the antithesis ‘ $\vdash x$ is measured by nothing’

K21) Theorem of the LT

- $\vdash \nabla y Wxy$ Measure
- Demonstration
- 1 $\vdash \nabla y Mxy$ Measure
 - 2 $\vdash Wxy \leftrightarrow Mxy$ Measure
 - 3 $\vdash \Delta y (Wxy \leftrightarrow Mxy)$ 2 Introduction of the Generalizing Assertion
 - 4 $\vdash \nabla y Wxy \leftrightarrow \nabla y Mxy$ 3 Distribution of the Bound Assertion
 - 5 $\vdash \nabla y Wxy$ 4,1 Separation

K22) The antithesis

Refutation

- 1 $\vdash \nabla y Wxy$ Measure
 - ┌ 2 $\vdash \neg \nabla y Wxy$ Hypothetical Premise (**Antithesis**)
 - ├ 3 $\vdash \nabla y Wxy \wedge \neg \nabla y Wxy$
 - ├ 4 $\vdash f$ 1,2 Logical Product
 - ├ 5 f 3 Logical Product
 - └ 6 $\vdash \neg (\vdash \neg \nabla y Wxy)$ 4 Elimination of the Assertion
- 2-5 Reduction to Absurdity

K3) The synthesis of ‘ $\vdash x$ is measured by something’ and ‘ $\vdash x$ is not measured by something’

Thesis	$\vdash \Delta y Wxy$	$\vdash x$ (existent or inexistent) is measured by everything (existent or inexistent).
Antithesis	$\vdash \neg \nabla y Wxy$	$\vdash x$ (existent or inexistent) is measured by nothing (existent or inexistent).
Synthesis (–) (T)	$\vdash \neg (\vdash \Delta y Wxy)$	\vdash Non ($\vdash x$ (existent or inexistent) is measured by everything (existent or inexistent).
Synthesis (–) (A)	$\vdash \neg (\vdash \neg \nabla y Wxy)$	\vdash Non ($\vdash x$ (existent or inexistent) is measured by nothing (existent or inexistent).
Synthesis (+) (T)	$\vdash \nabla y \neg Wxy$	$\vdash x$ (existent or inexistent) is not measured by something (existent or inexistent).
Synthesis (+) (A)	$\vdash \nabla y Wxy$	$\vdash x$ (existent or inexistent) is measured by something (existent or inexistent).

K4) Some apparently paradoxical developments of the synthesis

		K421	$\vdash Wxx$ $\vdash x$ is measured by x .
K412	$\vdash \forall y \neg Wxy$ $\vdash x$ is not measured by something.	K422	$\vdash \forall y Wxy$ $\vdash x$ is measured by something.
K413	$\vdash \Delta x \forall y \neg Wxy$ \vdash Everything is not measured by something.	K423	$\vdash \Delta x \forall y Wxy$ \vdash Everything is measured by something.
K414	$\vdash \forall x \forall y \neg Wxy$ \vdash Something is not measured by something.	K424	$\vdash \forall x \forall y Wxy$ \vdash Something is measured by something.
K415	$\vdash \neg \forall x \Delta y Wxy$ \vdash Nothing is measured by everything.	K425	$\vdash \neg \forall x \Delta y \neg Wxy$ \vdash Nothing is not measured by everything.
K416	$\vdash \neg \Delta x \Delta y Wxy$ \vdash Not everything is measured by everything.	K426	$\vdash \neg \Delta x \Delta y \neg Wxy$ \vdash Not everything is not measured by everything.

L) Measurability: the (com)measurable and the i(ncom)measurable

L1) Refutation of the thesis ‘ $\vdash x$ is of bad measurability’

L11) Additional Definitions in the LT

Postulation

- $Gm x \Leftrightarrow \forall y Wxy$ Good Measurability
[$Gm x$: x is of good measurability.]
- $Gim x \Leftrightarrow \forall y \neg Wxy$ Good Immeasurability
[$Gim x$: x is of good immeasurability.]
- $Bm x \Leftrightarrow \neg Gm x$ Bad Measurability
[$Bm x$: x is of bad measurability.]
- $Bim x \Leftrightarrow \neg Gim x$ Bad Immeasurability
[$Bim x$: x is of bad immeasurability.]

L12) Conceptual Correspondences

Development

$Gm x$	$\neg Bm x$	$\forall y Wxy$	$\neg \Delta y \neg Wxy$	Synthesis (A)
$Gim x$	$\neg Bim x$	$\forall y \neg Wxy$	$\neg \Delta y Wxy$	Synthesis (T)
$Bm x$	$\neg Gm x$	$\neg \forall y Wxy$	$\Delta y \neg Wxy$	Antithesis
$Bim x$	$\neg Gim x$	$\neg \forall y \neg Wxy$	$\Delta y Wxy$	Thesis

L13) Theorem of the LT

- $\vdash Gim x$ Good Immeasurability
- Demonstration
- 1 $\vdash \forall y \neg Wxy$ Measure [LT]
- 2 $\vdash Gim x$ 1 Good Immeasurability

L14) The thesis

Refutation

- 1 $\vdash GImm\ x$ Good Immeasurability [LT]
 - ┌ 2 $\vdash BImm\ x$ Hypothetical Premise (**Thesis**)
 - ├ 3 $\vdash \neg GImm\ x$ 1 Bad Immeasurability
 - └ 4 $\vdash GImm\ x \wedge \neg GImm\ x$
 - ┌ 5 $\vdash f$ 1,3 Logical Product
 - ├ 6 f 4 Supplementation
 - └ 7 $\vdash \neg (\vdash BImm\ x)$ 5 Elimination of the Assertion
- 1-6 Reduction to Absurdity

L2) Refutation of the antithesis ‘ $\vdash x$ is of bad measurability’

L21) Theorem of the LT

$\vdash G M m\ x$ Good Measurability

Demonstration

- 1 $\vdash \forall y\ Txy$ Introduction of the Transformation [LBA]
- 2 $\vdash G M m\ x$ 1 Good Measurability

L22) The antithesis

Refutation

- 1 $\vdash G M m\ x$ Good Measurability [LT]
 - ┌ 2 $\vdash B M m\ x$ Hypothetical Premise (**Antithesis**)
 - ├ 3 $\vdash \neg G M m\ x$ 2 Bad Measurability
 - └ 4 $\vdash G M m\ x \wedge \neg G M m\ x$
 - ┌ 5 $\vdash f$ 1,3 Logical Product
 - ├ 6 f 4 Supplementation
 - └ 7 $\vdash \neg (\vdash B M m\ x)$ 5 Elimination of the Assertion
- 2-6 Reduction to Absurdity

L3) The synthesis of ‘ $\vdash x$ is of good immeasurability’ and ‘ $\vdash x$ is of good measurability’

Thesis	$\vdash BImm\ x$	$\vdash x$ (existent or inexistent) is of bad immeasurability.
Antithesis	$\vdash BMm\ x$	$\vdash x$ (existent or inexistent) is of bad measurability.
Synthesis (-) (T)	$\vdash \neg (\vdash BImm\ x)$	\vdash Non ($\vdash x$ (existent or inexistent) is of bad immeasurability).
Synthesis (-) (A)	$\vdash \neg (\vdash BMm\ x)$	\vdash Non ($\vdash x$ (existent or inexistent) is of bad measurability).
Synthesis (+) (T)	$\vdash GImm\ x$	$\vdash x$ (existent or inexistent) is of good immeasurability.
Synthesis (+) (A)	$\vdash GMm\ x$	$\vdash x$ (existent or inexistent) is of good measurability.

L4) Some apparently paradoxical developments of the synthesis

L411	$\vdash GImm\ x$ $\vdash x$ is of good immeasurability.	L421	$\vdash GMm\ x$ $\vdash x$ is of good measurability.
L412	$\vdash \Delta x\ GImm\ x$ \vdash Everything is of good immeasurability.	L422	$\vdash \Delta x\ GMm\ x$ \vdash Everything is of good measurability.
L413	$\vdash \forall x\ GImm\ x$ \vdash Something is of good immeasurability.	L423	$\vdash \forall x\ GMm\ x$ \vdash Something is of good measurability.
L414	$\vdash \neg \forall x\ BImm\ x$ \vdash Nothing is of bad immeasurability.	L424	$\vdash \neg \forall x\ BMm\ x$ \vdash Nothing is of bad measurability.
L415	$\vdash \neg \Delta x\ BImm\ x$ \vdash Not everything is of bad immeasurability.	L425	$\vdash \neg \Delta x\ BMm\ x$ \vdash Not everything is of bad measurability.

M) Synopses

M1) Ideas, Postulates, Theses, Antitheses and Syntheses

‘X (Y)’ means ‘X (intimately related with Y)’.

IDEAS	POSTULATES	THESES & ANTI-THESES	SYNTHESES (T)	SYNTHESES (A)
A BOUND ASSERTION	[LBA]			
A' IDENTITY				
B TRANSFORMATION	$\vdash Txy \rightarrow x \neq y$ $\vdash \forall y Txy$	$\vdash \Delta y Txy$ $\vdash \neg \forall y Txy$	$\vdash \neg Txx$ $\vdash \forall y \neg Txy$	$\vdash \forall y Txy$
C (A') OTHERNESS	$x \neq y \Leftrightarrow \neg x = y$	$\vdash \Delta y x = y$ $\vdash \neg \forall y x = y$	$\vdash \forall y x \neq y$	$\vdash x = x$ $\vdash \forall y x = y$
D ACTIVE LIMITATION	$Lxy \Leftrightarrow x \neq y$	$\vdash \Delta y Lxy$ $\vdash \neg \forall y Lxy$	$\vdash \neg Lxx$ $\vdash \forall y \neg Lxy$	$\vdash \forall y Lxy$
E (D) PASSIVE LIMITATION	$\Gamma xy \Leftrightarrow Lyx$	$\vdash \Delta y \Gamma xy$ $\vdash \neg \forall y \Gamma xy$	$\vdash \neg \Gamma xx$ $\vdash \forall y \neg \Gamma xy$	$\vdash \forall y \Gamma xy$
F (E) GOOD FINITUDE, GOOD INFINITY, ...	$GFin x \Leftrightarrow \forall y \Gamma xy$ $GInf x \Leftrightarrow \forall y \neg \Gamma xy$ $BFin x \Leftrightarrow \neg GFin x$ $BInf x \Leftrightarrow \neg GInf x$	$\vdash BInf x$ $\vdash BFin x$	$\vdash \neg \Gamma xx$ $\vdash \forall y \neg \Gamma xy$ $\vdash GInf x$	$\vdash \forall y \Gamma xy$ $\vdash GFin x$
G (C) GOOD UNITY, GOOD PLURALITY, ...	$GUn x \Leftrightarrow \forall y x = y$ $GPl x \Leftrightarrow \forall y x \neq y$ $BUn x \Leftrightarrow \neg GUn x$ $BPl x \Leftrightarrow \neg GPl x$	$\vdash BPl x$ $\vdash BUn x$	$\vdash \forall y x \neq y$ $\vdash GPl x$	$\vdash x = x$ $\vdash \forall y x = y$ $\vdash GUn x$
H (B) GOOD MOBILITY, GOOD IMMOBILITY, ...	$GMob x \Leftrightarrow \forall y Txy$ $GImb x \Leftrightarrow \forall y \neg Txy$ $BMob x \Leftrightarrow \neg GMob x$ $BImb x \Leftrightarrow \neg GImb x$	$\vdash BImb x$ $\vdash BMob x$	$\vdash \neg Txx$ $\vdash \forall y \neg Txy$ $\vdash GImb x$	$\vdash \forall y Txy$ $\vdash GMob x$
I (B) (C) (D) (E) GOOD QUALITY, GOOD QUANTITY, ...	$GQl x \Leftrightarrow \forall y \neg Txy \wedge \forall y x \neq y$ $GQt x \Leftrightarrow \forall y Txy$ $BQl x \Leftrightarrow \neg GQl x$ $BQt x \Leftrightarrow \neg GQt x$	$\vdash BQl x$ $\vdash BQt x$	$\vdash \neg Txx$ $\vdash \forall y \neg Txy$ $\vdash \forall y x \neq y$ $\vdash GQl x$	$\vdash \forall y Txy$ $\vdash x = x$ $\vdash \forall y x = y$ $\vdash GQt x$
J (A') ACTIVE MEASURE	$Mxy \Leftrightarrow x = y$	$\vdash \Delta y Mxy$ $\vdash \neg \forall y Mxy$	$\vdash \forall y \neg Mxy$	$\vdash Mxx$ $\vdash \forall y Mxy$
K (J) PASSIVE MEASURE	$Wxy \Leftrightarrow Myx$	$\vdash \Delta y Wxy$ $\vdash \neg \forall y Wxy$	$\vdash \forall y \neg Wxy$	$\vdash Wxx$ $\vdash \forall y Wxy$
L (K) GOOD MEASURABILITY, GOOD IMMENSURABILITY, ...	$GMM x \Leftrightarrow \forall y Wxy$ $GImm x \Leftrightarrow \forall y \neg Wxy$ $BMM x \Leftrightarrow \neg GMM x$ $BImm x \Leftrightarrow \neg GImm x$	$\vdash BImm x$ $\vdash BMM x$	$\vdash \forall y \neg Wxy$ $\vdash GImm x$	$\vdash Wxx$ $\vdash \forall y Wxy$ $\vdash GMM x$

M2) Developments of Conceptual Correspondences F12) Between Good and Bad Finitudes and Infinitudes

$GFin\ x$	$\neg BFin\ x$	$\forall y\ \Gamma xy$	$\neg \Delta y \neg \Gamma xy$	Synthesis (A)
$GInf\ x$	$\neg BInf\ x$	$\forall y \neg \Gamma xy$	$\neg \Delta y \Gamma xy$	Synthesis (T)
$BFin\ x$	$\neg GFin\ x$	$\neg \forall y\ \Gamma xy$	$\Delta y \neg \Gamma xy$	Antithesis
$BInf\ x$	$\neg GInf\ x$	$\neg \forall y \neg \Gamma xy$	$\Delta y \Gamma xy$	Thesis

G12) Between Good and Bad Unities and Pluralities

$GUn\ x$	$\neg BUn\ x$	$\forall y\ x = y$	$\neg \Delta y\ x \neq y$	Synthesis (A)
$GPl\ x$	$\neg BPl\ x$	$\forall y\ x \neq y$	$\neg \Delta y\ x = y$	Synthesis (T)
$BUn\ x$	$\neg GUn\ x$	$\neg \forall y\ x = y$	$\Delta y\ x \neq y$	Antithesis
$BPl\ x$	$\neg GPl\ x$	$\neg \forall y\ x \neq y$	$\Delta y\ x = y$	Thesis

H12) Between Good and Bad Mobilities and Immobilities

$GMob\ x$	$\neg BMob\ x$	$\forall y\ Txy$	$\neg \Delta y \neg Txy$	Synthesis (A)
$GImb\ x$	$\neg BImb\ x$	$\forall y \neg Txy$	$\neg \Delta y Txy$	Synthesis (T)
$BMob\ x$	$\neg GMob\ x$	$\neg \forall y\ Txy$	$\Delta y \neg Txy$	Antithesis
$BImb\ x$	$\neg GImb\ x$	$\neg \forall y \neg Txy$	$\Delta y Txy$	Thesis

I16) Between Good and Bad Quantities and Qualities

1	$GQt\ x$	$\neg BQt\ x$	$\forall y\ Txy$	Synthesis (A)
2	$GQl\ x$	$\neg BQl\ x$	$\forall y \neg Txy \wedge \forall y\ x \neq y$	Synthesis (T)
3	$BQt\ x$	$\neg GQt\ x$	$\neg \forall y\ Txy$	Antithesis
4	$BQl\ x$	$\neg GQl\ x$	$\Delta y\ Txy \vee \Delta y\ x = y$	Thesis

1	$\forall y \neg Txy \wedge \forall y\ x \neq y \wedge \forall y \neg Lxy \wedge \forall y \neg \Gamma xy$	Synthesis (A)
2	$\forall y\ Txy \wedge \forall y\ x = y \wedge \forall y\ Lxy \wedge \forall y\ \Gamma xy$	Synthesis (T)
3	$\Delta y\ Txy \vee \Delta y\ x = y \vee \Delta y\ Lxy \vee \Delta y\ \Gamma xy$	Antithesis
4	$\neg \forall y\ Txy \vee \neg \forall y\ x = y \vee \neg \forall y\ Lxy \vee \neg \forall y\ \Gamma xy$	Thesis

L12) Between Good and Bad Measurabilities and Immeasurabilities

$GMM\ x$	$\neg BMM\ x$	$\forall y\ Wxy$	$\neg \Delta y \neg Wxy$	Synthesis (A)
$GImm\ x$	$\neg BImm\ x$	$\forall y \neg Wxy$	$\neg \Delta y Wxy$	Synthesis (T)
$BMM\ x$	$\neg GMM\ x$	$\neg \forall y\ Wxy$	$\Delta y \neg Wxy$	Antithesis
$BImm\ x$	$\neg GImm\ x$	$\neg \forall y \neg Wxy$	$\Delta y Wxy$	Thesis

M3) General Scheme for Unreflexive Relations

P: Premise HP: Hypothetical Premise RA: Reduction to Absurdity

<p style="text-align: center;">THESIS</p> $\begin{array}{l} \vdash \neg Rxx \quad P \\ \vdash \forall y \neg Rxy \\ \quad \lceil \vdash \Delta y Rxy \quad HP \\ \quad \dots \dots \dots \\ \quad \lfloor f \\ \vdash \neg (\vdash \Delta y Rxy) \quad RA \end{array}$	<p style="text-align: center;">ANTITHESIS</p> $\begin{array}{l} \vdash \forall y Rxy \quad P \\ \quad \lceil \vdash \neg \forall y Rxy \quad HP \\ \quad \dots \dots \dots \\ \quad \lfloor f \\ \vdash \neg (\vdash \neg \forall y Rxy) \quad RA \end{array}$
SYNTHESIS	
Positive Part	
<p style="text-align: center;">Logical Law Refuting the Thesis</p> $\vdash \forall y \neg Rxy$	<p style="text-align: center;">Logical Law Refuting the Antithesis</p> $\vdash \forall y Rxy$
Negative Part	
<p style="text-align: center;">Metalogical Law of the Refuted Thesis</p> $\vdash \neg (\vdash \Delta y Rxy)$	<p style="text-align: center;">Metalogical Law of the Refuted Antithesis</p> $\vdash \neg (\vdash \neg \forall y Rxy)$

M4) General Scheme for Reflexive Relations

P: Premise HP: Hypothetical Premise RA: Reduction to Absurdity

<p style="text-align: center;">THESIS</p> $\begin{array}{l} \vdash \forall y \neg Rxy \quad P \\ \quad \lceil \vdash \Delta y Rxy \quad HP \\ \quad \dots \dots \dots \\ \quad \lfloor f \\ \vdash \neg (\vdash \Delta y Rxy) \quad RA \end{array}$	<p style="text-align: center;">ANTITHESIS</p> $\begin{array}{l} \vdash Rxx \quad P \\ \vdash \forall y Rxy \\ \quad \lceil \vdash \neg \forall y Rxy \quad HP \\ \quad \dots \dots \dots \\ \quad \lfloor f \\ \vdash \neg (\vdash \neg \forall y Rxy) \quad RA \end{array}$
SYNTHESIS	
Positive Part	
<p style="text-align: center;">Logical Law Refuting the Thesis</p> $\vdash \forall y \neg Rxy$	<p style="text-align: center;">Logical Law Refuting the Antithesis</p> $\vdash \forall y Rxy$
Negative Part	
<p style="text-align: center;">Metalogical Law of the Refuted Thesis</p> $\vdash \neg (\vdash \Delta y Rxy)$	<p style="text-align: center;">Metalogical Law of the Refuted Antithesis</p> $\vdash \neg (\vdash \neg \forall y Rxy)$